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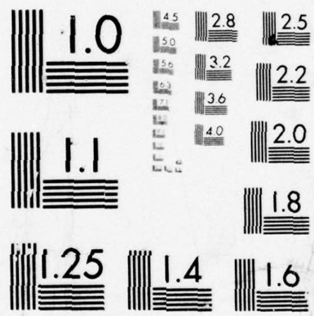

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TURBULENT PRESSURE SPECTRA

W. K. George, Jr., P. D. Beuther, R. E. A. Arndt

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The Pennsylvania State University
Institute for Science and Engineering
APPLIED RESEARCH LABORATORY
Post Office Box 30
State College, PA 16801

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Introduction

The idea of an equilibrium range in the turbulent energy spectrum was first proposed by Kolmogorov (1941 a,b) who argued that the parts of the spectrum whose time scales were much smaller than that for overall decay might exist in statistical equilibrium (c.f. Batchelor [1953]). The only characteristic parameters for this equilibrium range are the viscosity, ν, and the rate of transfer of spectral energy, ε. The existence of such a spectral equilibrium range was shown to be exptected when the turbulence Reynolds number, $u\,\ell/\nu$, was much greater than one;* in particular, $(u\,\ell/\nu)^{3/4} >> 1$ (c.f. Batchelor [1953], Tennekes and Lumley [1972]).

For sufficiently large separation of the energy containing range (the range dominating the decay dynamics) and the dissipation range it was further postulated that an inertial subrange should exist depending only on the rate of transfer of spectral energy, ε. The existence of the inertial subrange is contingent on even larger turbulent Reynolds numbers than before; $(\frac{u\,\ell}{\nu})^{3/8} >> 1$ (c.f. Tennekes and Lumley [1972], Batchelor [1953]).

These ideas were applied with great success to a variety of turbulent flow properties. The best known result among these is, of course, that for the turbulent velocity spectrum where the form of the spectrum in the inertial subrange was shown to be given by

$$E(k) = \alpha\,\varepsilon^{2/3}\,k^{-5/3} \qquad (1)$$

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* $u^2 \equiv 1/3(u_1^2 + u_2^2 + u_3^2)$

where E(k) is the three-dimensional energy spectrum obtained by averaging the directional spectrum over spherical shells of radius k, k is the wavenumber, and α is a 'universal' constant. As Kraichnan (1974) has observed, the spectral predictions from universal equilibrium theory have had "an embarrassment of success", even in cases where they would not have been expected to work. While the universal equilibrium arguments are strictly valid for only homogeneous flows, the use of the local value of ε appears to extend the validity of the arguments to a variety of inhomogeneous flow situations even when the scale of the inhomogeneity is the same size as the energy containing eddies.

In 1953, Batchelor commented that the pressure spectrum "$\pi(k)$ is readily found to be $\pi(k) \sim k^{-7/3}$ for $k >> 1/\ell$". This comment appears to have been almost universally unnoticed by many recent investigators of the turbulent pressure field. Moreover, the entire subject of similarity scaling for the pressure spectrum has not been broached in spite of adequate experimental data.

In this paper the similarity scaling laws for the pressure spectrum at both high and low wavenumber are reviewed. Following Tennekes and Lumley [1972], the $k^{-7/3}$ range is seen to result from an asymptotic matching of the high and low wavenumber similarity spectra as the Reynolds number becomes large. Predictions are compared with recent experimental data and the universal constant is evaluated.

The Small Scale Spectrum (large k)

For wavenumbers large compared to that characterizing the energy containing range we expect the universal equilibrium theory to apply. Accordingly, ν, ε and ρ must be the major parameters and we have $\pi=\pi(k,\varepsilon,\nu,\rho)$. The only possible dimensionless form is

$$\frac{\pi(k)}{\rho^2\nu^{7/4}\varepsilon^{3/4}} = \frac{\pi(k)}{\rho^2\upsilon^2\eta} = f_{pp}(k\eta) \tag{2}$$

where η is the Kolmogorov microscale, $\eta=(\nu^3/\varepsilon)^{1/4}$, and where υ is the Kolmogorov velocity $\upsilon=(\nu\varepsilon)^{1/4}$.

In view of the local isotropy of small scale turbulence and the success of the universal equilibrium scaling for the velocity spectrum we expect Equation (2) to be valid for a large variety of turbulent flows.

If the turbulent Reynolds number is sufficiently high, we expect that there exists an inertial subrange; that is, a region in which the only major parameter is ε. Thus we have $\pi=\pi(\varepsilon,k,\rho)$ and the only possible dimensionless combination yields

$$\pi(k) = \alpha_p\rho^2\varepsilon^{4/3}k^{-7/3} \quad . \tag{3}$$

This was first obtained by Batchelor [1953].

If in the inertial subrange we associate the pressure fluctuation at a given wavenumber with the velocity at that wavenumber we can obtain a relation between α_p and the corresponding coefficient for the velocity spectrum. We take $u_k \sim \varepsilon^{1/3}k^{-1/3}$ on dimensional grounds where u_k is the velocity of "eddies" of size 1/k. The turbulent energy spectrum is then given by

$$E(k) = \frac{u_k^2}{k}$$

$$= \alpha\,\varepsilon^{2/3}\,k^{-5/3} \quad . \tag{4}$$

It follows that

$$u_k = \sqrt{\alpha}\ \varepsilon^{1/3}\ k^{-1/3} \quad . \tag{5}$$

The pressure spectrum is given by

$$\pi(k) = \frac{\rho^2 u_k{}^4}{k} \tag{6a}$$

$$= \alpha_p\ \varepsilon^{4/3}\ k^{-7/3} \tag{6b}$$

from which it follows that

$$\alpha_p = \alpha^2 \quad . \tag{7}$$

For *moderate* Reynolds numbers $\alpha \simeq 1.5$ and therefore we expect $\alpha_p \simeq 2.25$.

The reasoning above can be extended to derive an analytical form for the entire wavenumber spectrum in the universal equilibrium range. One of the most useful forms of the velocity spectrum that has been proposed is due to Pao (1965) and is given by

$$E(k) = \alpha\ \varepsilon^{2/3}\ k^{-5/3}\ \exp\ [-(3/2)\ \alpha(k\eta)^{4/3}] \quad . \tag{8}$$

Using this for u_k in Equation (4) and substituting into Equation (6a) we obtain

$$\frac{\pi(k)}{\rho^2 \varepsilon^{3/4} \nu^{7/4}} = \alpha^2 (k\eta)^{-7/3}\ \exp\ [-3\alpha(k\eta)^{4/3}] \quad . \tag{9}$$

This form for the pressure spectrum is shown in Figure (1).

The deficiencies of Pao's spectrum near $k \sim 1/\eta$ are well known and are equally applicable here. Equation (9) does have the advantage that it is of closed form and readily integrable.

The Large Scale Spectrum

The large scale spectrum be definition is dominated by those turbulent eddies which contain most of the turbulent energy and whose characteristic times are the same order as the decay time. Since the process by which energy is added to the turbulence is geometry dependent we should not expect a universal scaling as in the equilibrium range but at best similarity spectra which are valid only for a particular class of flows. On dimensional grounds $\overline{p^2}$, the local mean square turbulent pressure fluctuation, and ℓ, a characteristic length for the turbulence, might provide a useful scaling. We have

$$\frac{\pi(k\ell)}{\ell \cdot \overline{p^2}} = F_{pp}(k\ell) \quad . \tag{10}$$

The characteristic length, ℓ, remains to be specified. Since we are seeking a length characteristic of the decay process, we will choose the length defined by $\varepsilon = u^3/\ell$ where ε is the local rate of decay of turbulent energy and u is the local rms turbulent velocity.

In an incompressible flow the pressure is given by

$$\nabla^2 \, p(\underset{\sim}{x},t) = -\rho \, \frac{\partial^2 u_i' u_j'}{\partial x_i \partial x_j} \tag{11}$$

where u_i' is the total velocity vector. This can be solved to yield

$$p(\underset{\sim}{x},t) = \frac{\rho}{4\pi} \int_{\substack{\text{all}\\ \text{space}}} \frac{\partial^2 u_i' u_j'}{\partial y_i \partial y_i} \frac{1}{|\underset{\sim}{y}-\underset{\sim}{x}|} d^3 y \quad . \tag{12}$$

It is clear that the pressure depends on not only the flow at the point in question but on the flow at a distance. Thus, there is no reason, a priori, we should expect that the length ℓ will accomplish the scaling we seek. It might be noted here that the same considerations are valid for the turbulent energy spectrum where ε must be reasonably constant over distance of at least ℓ if the large scale turbulence structure is to be scaled.

In general, more information is known about the turbulent velocity field than about the turbulent pressure field. It would be very useful if the pressure spectra could be scaled in terms of velocity parameters alone instead of $\overline{p^2}$. We propose similarity relations for the large scale spectrum based on ρ, u, and ℓ and attempt to indicate where the relations might be useful. From dimensional considerations we can write immediately for the large scale spectrum

$$\frac{\pi(k)}{\rho^2 u^4 \cdot \ell} = F_{pp}(k\ell) \quad . \tag{13}$$

To examine the range of validity of Equation (13) we will look at the dependence of the turbulent pressure field on the turbulent velocity field.

For a homogeneous, isotropic turbulence Batchelor [1953] was able to show that

$$\pi(k) = \frac{1}{8\pi^2} \int_{\infty} E(k')E(|\underset{\sim}{k}-\underset{\sim}{k}'|) \frac{\sin^4\Theta}{|\underset{\sim}{k}-\underset{\sim}{k}'|^4} dk' \quad . \tag{14}$$

This relation was based on an assumption of normality for fourth order moments and is not expected to be valid for high wavenumbers. From Equation (14) Batchelor argued that the principal contribution to $\overline{p^2}$ came from wavenumbers near the maximum of E(k), the velocity spectrum. Since the maximum of E(k) is at about $1/\ell$ we should expect Equation (13) to work for this restricted class of turbulent flows.

Kraichnan [1956] treated a slightly more general class of flows than Batchelor and included the effects of anisotropy and a uniform shear. The effect of anisotropy was found to weaken slightly the mean square pressure fluctuation for a given turbulent energy. For the case of an isotropic, homogeneous turbulence acted upon by a uniform and steady mean shear, Kraichnan showed that there were two contributions to the mean square pressure, a contribution from the turbulence alone, $\overline{p^2}_{T-T}$, and a contribution from the turbulence-shear interaction, $\overline{p^2}_{T-S}$. Under the assumption of normality for fourth order moments and an exponentially decaying spatial correlation for the velocity field, these terms were found to be

$$\overline{p^2}_{T-T} \sim (1/2\rho\overline{u^2})^2$$

and

$$\overline{p^2}_{T-S} \sim \rho^2\overline{u^2}\ell^2 \left(\frac{d\bar{u}}{dy}\right)^2 \quad . \tag{15}$$

Arndt, et al. [1974] and others have observed reasonably good agreement with these predictions in jet mixing layers. The predictions

of Equation (15), however, do not seem adequate in the adjustment region of jets or in more general classes of flows such as a body-perturbed jet.

As noted by Kraichnan, isotropic homogeneous turbulence subjected to steady uniform mean shear is a highly artificial example since such a flow would quickly evolve. The success of Kraichnan's theory in mixing layers is probably due to the fact that the turbulence is always in local scale equilibrium; that is the ratio $(\ell/u)(d\bar{u}/dy) \simeq$ constant. If this is the reason, then Kraichnan's result is forced by dimensional considerations and probably will not be valid in flows involving more than one time scale as in the wall region of a boundary layer or in a perturbed jet. This appears to be the case. It is interesting to speculate whether Kraichnan's results could be extended to a more general class of flows by using Lumley's [1967a] concepts of limited spatial awareness and limited temporal memory.

In flows which are characterized by a single time and length scale we expect Equation (13) to provide scaling. For more general flows, it will be necessary to revert to the form proposed in Equation (10) which requires only that ℓ be the appropriate length.

The Turbulence-Turbulence Interaction Spectrum

To date no satisfactory method for experimentally separating the pressure fluctuations due to the turbulence-turbulence interaction and the turbulence-mean shear interaction has been proposed. In the paragraphs below a spectral form for the turbulence-turbulence pressure spectra is proposed. It is suggested that spectral deviations from this form represent the turbulence-mean shear interaction.

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By assuming that the work done by the mean strain rate per unit wavenumber and per unit time is determined only by local quantities, Tennekes and Lumley [1972] obtained a low wavenumber velocity spectrum for turbulent flow:

$$E(k) = \alpha\, \varepsilon^{2/3}\, k^{-5/3} \exp\left[-3/2\pi\, \beta\, \alpha^{1/2} (k\ell)^{-4/3}\right] \tag{16}$$

where α is the Kolmogorov constant and ℓ is determined by $\varepsilon = u^3/\ell$. The spectrum was required to integrate to the kinetic energy $3/2\ \overline{u^2}$ which yielded $\beta=0.3$. The model was not expected to be valid for $k\ell \ll 1$.

Since the hypothesis of dependence on local parameters has already been made, we can apply arguments similar to those preceding Equation (9) and obtain for the low wavenumber spectrum

$$F_{pp}(k\ell) = \alpha^2\, (k\ell)^{-7/3} \exp\left[-3\pi\, \beta\, \alpha^{1/2}\, (k\ell)^{-4/3}\right] \tag{17}$$

or

$$\pi(k) = \alpha^2 \rho^2 \varepsilon^{4/3}\, (k\ell)^{-7/3} \exp\left[-3\pi\, \beta\, \alpha^{1/2}\, (k\ell)^{-4/3}\right] \quad .$$

This is shown graphically in Figure (2).

The Inertial Subrange

The dependence of the existence of an inertial subrange on turbulent Reynolds number can best be illustrated by a technique used by Tennekes and Lumley [1972] for the velocity spectrum. We have proposed similarity laws for the spectrum at both high and low wavenumber. As the turbulent Reynolds number becomes large, the energy containing range and the dissipation range become more widely separated. Therefore, we ask whether there

exists a region of common validity for both spectral forms if we take the limits $R\to\infty$, $k\ell\to\infty$, and $k\eta\to 0$ simultaneously.

It is well known that $\eta\ell \sim R^{-3/4}$ (Tennekes and Lumley [1972]). To take the limits we let $k\ell \sim R^{n}$ where $n>0$. In order for $k\eta\to 0$ as $R\to\infty$ we must have $k = k\ell(\eta/\ell) \sim R^{n-3/4} \to 0$. Therefore, any n satisfying $0<n<3/4$ will suffice.

Substituting these relations into Equations (2) and (13) we obtain

$$\rho^2 \varepsilon^{3/4} \nu^{7/4} f_{pp}(R^{n-3/4}) = \rho^2 u^4 \ell \, F_{pp}(R^n) \tag{18}$$

which, using $\varepsilon = u^3/\ell$, becomes

$$f_{pp}(R^{n-3/4}) = R^{7/4} F_{pp}(R^n) \quad . \tag{19}$$

This must be satisfied for any n in the interval $0<n<3/4$. The solution is readily obtained as

$$\begin{aligned} f_{pp}(k\eta) &= \alpha_p (k\eta)^{-7/3} \\ F_{pp}(k\eta) &= \alpha_p (k\ell)^{-7/3} \quad . \end{aligned} \tag{20}$$

Thus there is a region of overlap and it corresponds to the equilibrium range found previously.

<u>The Mean Square Pressure Fluctuation</u>

The mean square pressure fluctuation is obtained from the spectrum by

$$\overline{p^2} = \frac{\rho^2\overline{u^2}\cdot\ell}{\ell}\int_0^\infty F_{pp}(k\ell)\,dk\ell \quad . \tag{22}$$

For, geometrically similar flows the integral is nearly a constant and we can write

$$\overline{p^2} = A_1 \cdot \rho^2\overline{u^2}^2 \quad . \tag{23}$$

The slight dependence of A_1 on Reynolds number results from the increasing extent of the $(k\ell)^{-7/3}$ range with Reynolds number when the low wavenumber scaling is applied.

A value for A_1 can be obtained for high Reynolds numbers by integrating the spectral form of Equation (17).

$$\overline{p^2} = \rho^2\varepsilon^{4/3}\ell^{4/3}\alpha^2\int_0^\infty (k\ )^{-7/3}\exp\left[-3\pi\ \beta\ \alpha^{1/2}\ (k\ell)^{-4/3}\right] dk\ell$$

$$\cong \frac{\alpha^{3/2}}{4\pi\beta}\,\varepsilon^{4/3}\ell^{4/3}\rho^2 \quad . \tag{24}$$

Using α=1.5 and ε=u^3/ℓ we obtain

$$\overline{p^2} \cong 0.487\rho^2\overline{u^2}^2 \quad . \tag{25}$$

Hinze [1959] calculated p' from an assumed form of the velocity correlation $f(r) = \overline{u_1(x)u_1(x+r)}$ for isotropic turbulence. Using simple exponential decay (high Reynolds number) he obtained $\overline{p^2} \simeq 0.49\rho^2\overline{u^2}^2$ and by using a Gaussian correlation (low Reynolds number) $\overline{p^2} \simeq 1.0\rho^2\overline{u^2}^2$. Batchelor [1951] used the velocity correlation measured in a grid turbulence

to obtain $\overline{p^2} \simeq 0.34\ \rho\ \overline{u^2}^2$. The estimate obtained here and the high Reynolds estimate of Hinze are in excellent agreement with the results of Uberoi [1954] who obtained $p' \simeq 0.7\ \rho u'^2$. This agreement with Uberoi's results is particularly gratifying for three reasons: first, there are no adjustable constants in this theory; second, Uberoi's results were obtained in a nearly isotropic turbulence where no turbulence shear interaction term exists; and third, the results were inferred from velocity measurements, thus avoiding the question as to whether the probe was measuring pressure fluctuations.

The Mean Square Fluctuating Pressure Gradient

The mean square fluctuating pressure gradient is given by

$$\overline{(\nabla p)^2} = \int_0^\infty k^2\ \pi(k)\ dk \quad . \qquad (26)$$

The integrand will rise rapidly for low wavenumbers and peak near the low wavenumber end of the inertial subrange beyond which it will roll off as $k^{-1/3}$. Thus most of the contribution will come from the $k^{-1/3}$ range for high Reynolds number flow. It is interesting to note that this spectrum does not peak at high wavenumbers as does the velocity derivative spectrum.

To evaluate the mean square pressure gradient for large Reynolds number we use the high wavenumber spectrum of Equation (9) and obtain

$$\overline{(\nabla p)^2} = \rho^2(\varepsilon^3/\nu)^{1/2}\ \alpha^2 \int_0^\infty (k\eta)^{-1/3} \exp\left[-3\alpha\ (k\eta)^{4/3}\right] dk\eta \quad . \qquad (26)$$

Making the variable change $y = (k\eta)^{2/3}$ this can readily be integrated to yield

$$\overline{(\nabla p)^2} = \frac{\sqrt{3\pi}}{4}\, \alpha^{3/2} \rho^2\, (\varepsilon^3/\nu)^{1/2} \tag{27a}$$

or using $\alpha \simeq 1.5$ we obtain

$$\overline{(\nabla p)^2} \simeq 4.0\, \rho^2\, (\varepsilon^3/\nu)^{1/2} \quad . \tag{27b}$$

Batchelor [1951] obtained a similar result from an approximate spectral model.

The One Dimensional Spectrum

While the three-dimensional spectrum $\pi(k)$ is useful for dynamical considerations, it is the one-dimensional spectrum $F^1_{pp}(k_1)$ that is usually measured. Their relationship is described below.

The pressure covariance for homogeneous turbulent flow is defined by

$$R_{pp}(\underset{\sim}{r}) = \overline{p(\underset{\sim}{x})\; p(\underset{\sim}{x} + \underset{\sim}{r})} \tag{28}$$

and its Fourier transform, the pressure spectrum is

$$\Phi_{pp}\,(k_1,k_2,k_3) = \frac{1}{(2\pi)^3} \iiint_{-\infty}^{\infty} R_{pp}(\underset{\sim}{r})\; e^{-i\underset{\sim}{k}\cdot\underset{\sim}{r}}\; d\underset{\sim}{r} \quad . \tag{29}$$

The one-dimensional pressure spectrum $F^1_{pp}(k_1)$ is defined by

$$F^{1}_{pp}(k_1) = \iint_{-\infty}^{\infty} \Phi_{pp}(k_1,k_2,k_3)\; dk_2 dk_3 \quad . \tag{30}$$

The three dimensional spectrum $\pi(k)$ is obtained from $\Phi_{pp}(\underset{\sim}{k})$ by averaging over spherical shells of radius $\underset{\sim}{k}$; that is

$$\pi(k) = \frac{1}{4\pi k^2} \iint \Phi_{pp}(\underset{\sim}{k})\; d\sigma\;(k) \tag{31}$$

where $d\sigma(k)$ is an element of area on a sphere of radius $|\underset{\sim}{k}| = k$.

If the turbulence is isotropic there exists a relation between $\pi(k)$ and $F^{1}_{pp}(k_1)$ (c.f., Hinze [1959] and Lumley [1970]):

$$F^{1}_{pp}(k_1) = 1/2 \int_{k_1}^{\infty} \frac{\pi(k)}{k}\, dk \tag{32a}$$

or

$$\pi(k) = -2k \frac{\partial F^{1}_{pp}(k)}{\partial k} \quad . \tag{32b}$$

It is clear from Equation (32b) that a power law region of $\pi(k)$ will also appear as the same power law in $F^{1}_{pp}(k_1)$. Thus the one-dimensional velocity spectrum will also exhibit a $k^{-7/3}$ region. We obtain for the inertial subrange

$$\tilde{f}^{1}_{pp}(k_1) = 3/14\; \alpha_p\; \tilde{k}_1^{-7/3} \quad . \tag{33}$$

The one-dimensional pressure spectrum analog of Pao's velocity spectrum can be obtained by substituting Equation (9) into Equation (32a) and integrating. In dimensionless form

$$\tilde{f}^1_{pp}(\tilde{k}_1) = 1/2\ \alpha^2 \int_{k_1}^{\infty} \tilde{k}^{-10/3} \exp\left[-3\ \alpha\tilde{k}^{4/3}\right] d\tilde{k} \tag{34}$$

where

$$f^1_{pp} = \frac{f^1_{pp}}{\rho^2 \varepsilon^{3/4} \nu^{7/4}}$$

$$\tilde{k}_1 = k_1\eta \quad , \qquad \tilde{k} = k\eta \quad . \tag{35}$$

Following Pao [1965] we introduce the variable $\xi = k_1/k$ and obtain

$$\tilde{f}^1_{pp}\ (\tilde{k}_1) = 1/2\ \alpha^2\ \tilde{k}_1^{\ -7/3} \int \xi^{4/3} \exp\left[-3\alpha\ (k_1/\xi)^{4/3}\right] d\xi \quad . \tag{36}$$

This has been integrated numerically; the results are tabulated in Table I and shown graphically in Figure (3).

A similar exercise can be carried out for the large scale spectrum with the following integral resulting:

$$\frac{F^1_{pp}(k\ell)}{\rho^2 u'^4 \ell} = 1/2\ \alpha^2\ (k_1\ell)^{-7/3} \int_o^1 \xi^{4/3} \exp\left[-3\pi\beta\alpha^{1/2} \left(\frac{k_1}{\xi}\right)^{-4/3}\right] d\xi \quad . \tag{37}$$

This has also been integrated numerically; the results are tabulated in Table II and shown graphically in Figure (4).

It should be remembered that both of these results assumed isotropy and therefore their applicability to experimental data is limited to such flows. On the other hand, in view of the local isotropy of most high Reynolds number turbulent shear flows at small scales, the high wavenumber spectrum should find wide application.

The Experimental Results

An attempt was made to collapse the considerable body of data that has appeared in the literature in recent years (c.f., Planchon [1974], Fuchs [1972], Arndt, Tran, and Barefoot [1974]). Although all of these pressure spectral measurements indicated a -7/3 region, dimensionless scaling was not possible because estimates of u, ℓ, and ε were not available. Therefore an investigation was conducted in the mixing layer of a 12 inch open jet.

The unsteady pressure probe used in this investigation was developed by Arndt and Nilsen [1969]. The sensitive element is a Bruel and Kjaer 1/8 inch condenser microphone which is connected to a cathode follower and powered by a B & K Type 2801 Microphone Power Supply Unit. The probe is a standard pitot static tube, 0.12 inches outside diameter and 2.5 inches in length. Four static pressure holes are spaced 90 degrees apart at a distance of 5 tube diameters from the tip of the probe; the other end of the probe is terminated by the microphone. Damping material was placed in the tube to diminish a response peak at tube resonance frequency.

The probe was calibrated by placing it and a reference microphone in a reverberent chamber. The calibration and a sketch of the probe are shown in Figure (5). Spectra were measured using a Federal Scientific Real-Time Analyzer. The effective bandwidth was 10 Hz and the effective bandwidth-averaging time product was 400. Spectra were plotted on a x-y plotter as log spectrum versus frequency and the data points shown were read from these plots. This procedure introduced errors estimated at 20%. The measured spectra were corrected for the temporal response of the probe up to a maximum correction of 2dB. No attempt was made to correct for the

limited spatial response which was judged to be considerable at frequencies above $f=\overline{u}_c/10d$ where $\overline{u}_c$ is the convection velocity and 5d is the distance from the probe tip to the static holes.

Arndt, Tran, and Barefoot [1974] using the same probes report that at the center of the mixing layer

$$p' = 0.0625\ (1/2\rho\ \overline{u}_E^{2})$$

which corresponds to

$$p' = 1.26\ \rho\overline{u^2} \quad .$$

Hence for this flow it can be assumed that the shear interaction term is of the same order as the turbulence term. Thus, this experimental situation provides a good test for the scaling arguments of the previous sections.

Spectral measurements were made in the center of the mixing layer at locations of x/D = 1, 1.5, 2, 3 and 4 where D is the jet diameter. The exit velocities used were 19.8 m/s and 30.5 m/s. Since the exit diameter was 0.305 m, the exit Reynolds numbers were 4.0×10^5 and 6.2×10^5 respectively. The previous work of Arndt et al. [1974] indicated that the lower velocity was the smallest that could be used without appreciable contamination from background noise.

There have been several exhaustive series of measurements of velocity data in the axisymmetric jet mixing layer (c.f. Davies, et al. [1963], Bradshaw et al. [1964]). Unfortunately, the rate of dissipation of turbulent energy per unit volume, ε, was not measured. Because of the extremely small probes that would have been required to accurately determine the necessary velocity derivatives, these measurements were not

carried out. It is possible, however, to state the approximate functional dependence of u, ε and ℓ on the distance from the jet lip, x, and the exit velocity, U_E. These are readily obtained as

$$\ell \sim x$$

$$u \sim U_E$$

from which it follows that

$$\varepsilon = \frac{u^3}{\ell} \sim \frac{U_E^3}{x} \quad .$$

Thus the data obtained in this experiment are plotted with two nondimensionalizations corresponding to the high and low wavenumber forms discussed earlier:

$$\frac{f^1_{pp}}{\rho^2 \varepsilon^{3/4} \nu^{7/4}} \sim \frac{f^1_{pp}}{\rho^2 U_E^4 x} \left(\frac{U_E x}{\nu}\right) \qquad \text{high wavenumber}$$

$$k\eta \sim kx \left(\frac{\nu}{U_E x}\right)^{3/4}$$

and

$$\frac{F^1_{pp}}{\rho^2 u^4 \ell} \sim \frac{F^1_{pp}}{\rho^2 U_E^4 x} \qquad \text{low wavenumber}$$

$$k\ell \sim kx \quad .$$

The Kolmogorov-like spectra (high wavenumber scaling) are shown in Figure 6. Wavenumbers were calculated by using a fixed convection velocity equal to the mean velocity at the mixing layer center ($U_c = 0.6\ U_E$). In view of the accuracy of the results, application of Lumley's [1967b] correction to Taylor's hypothesis hardly seems justifiable (less than 20%).

In particular, the measurement of the mean velocity is accurate only to within 5%. Moreover, since measurements were taken at the center of the mixing layer where the mean velocity gradient is a maximum, additional error could result from a slight mislocation of the probe. Since velocity enters to the fourth power, the observed 20% scatter is not unreasonable. There is some evidence of such systematic error in the normalized plots.

The roll-off at high wavenumbers is attributed to the spatial averaging of the probe. The slight positive deviation from the -7/3 law deviation is probably due to background noise since it progressively disappears as the absolute spectral level increases. The line drawn through the data is chosen to fit the highest absolute data for which the background effect would be the least ($x/D = 1$, $U = 30.5$ m/s).

The low wavenumber plots of measured spectra are shown in Figure (7). It is clear that the inertial subrange also collapses on this plot. The low wavenumber data do not collapse and the spectral value is seen to decrease with distance from the origin. Certainly Taylor's hypothesis is not valid for wavenumbers in the range $k\ell \leq 1$ (c.f. Lumley [1967b]). In particular, the eddy turnover time or life-time is of the same order as the time required to pass the probe. Such behavior would also be observed if the pressure fluctuations resulting from the shear interaction were dominated by the large eddy structure (c.f. Arndt and George [1974]). Therefore this disparity at low wave numbers is not felt to contradict the low wavenumber scaling hypothesis.

Wygnanski and Fiedler [1970] measured the Taylor microscales in the plane mixing layer and obtained approximately $\lambda_f \simeq 4.6 \sqrt{\nu x/U_E}$.* Assuming

*Interpreted from Figure (37), opus cit.

local isotropy (which was not confirmed in the work cited) one obtains

$$\varepsilon \simeq 0.046 \frac{U_E^3}{x} \quad .$$

Using $u \simeq 0.16\ U_E$ from the same work and confirmed in this study, one obtains $\ell \simeq 0.08x$. When these values are substituted into the scaling laws and compared to the measured pressure spectra, the value of α_p is obtained as 5.6. If on the other hand ℓ is selected as $\ell \sim 0.04x$ the value suggested earlier can be obtained; i.e., $\alpha_p = 2.25$. In view of the difficulties associated with accurate dissipation measurements in such flow, the latter values does not seem unreasonable.

Figure (8) shows a plot of the pressure spectra normalized with exit parameters U_E, D and Strouhal number. It is clear that the pressure spectra maxima do not collapse in this type of plot and the spectral peak clearly shifts to lower frequencies with increasing x as would be expected.

In all plots it is clear that for the lowest frequencies the spectrum increases quadratically with increasing frequencies. The peak and slope is taken to indicate that the pressure space correlation is negative somewhere (c.f. Lumley [1970]). This interpretation is consistent with the observations of Planchon and Jones [1974].

The fact that the spectra display a limited -7/3 range is interpreted here as confirmation that the pressure probe is indeed measuring static pressure. If the probe were responding to stagnation pressure, the dominant fluctuating term would be the cross product of the mean and fluctuating velocity; thus a -5/3 spectrum would have been observed. This is perhaps the explanation for the data of Elliot, et al. [1972].

Summary and Conclusions

Scaling laws have been proposed for both high and low wavenumber pressure spectra. Exact spectral forms for both high and low wavenumbers have been derived. An inertial subrange behaving as $\pi(k) \sim k^{-7/3}$ has been shown to exist. Further measurements, especially of the turbulent dissipation, are necessary to evaluate the Kolmogorov constant although it appears to be between the value of 2.25 suggested and 5.6. Meaurements of pressure spectra in the mixing layer of an axisymmetric jet were shown to follow the predicted scaling laws at both high and low wavenumbers.

Table I

High Wavenumber One-Dimensional Spectrum (Eq. 36)

$k\eta$	$\frac{f^{1}_{pp}}{\rho^2 \varepsilon^{3/4} \nu^{7/4}}$	$k\eta$	$\frac{f'_{pp}}{\rho^2 \varepsilon^{3/4} \nu^{7/4}}$
1×10^{-3}	4.816×10^{6}	6×10^{-2}	2.768×10^{2}
2×10^{-3}	9.542×10^{5}	8×10^{-2}	1.294×10^{2}
		1×10^{-1}	7.001×10^{1}
4×10^{-3}	1.886×10^{5}	2×10^{-1}	8.374
6	7.289×10^{4}	4×10^{-1}	5.413×10^{-1}
8	3.706×10^{4}	6×10^{-1}	6.236×10^{-2}
1×10^{-2}	2.190×10^{4}	8×10^{-1}	8.794×10^{-3}
2×10^{-2}	4.210×10^{3}	1	1.353×10^{-3}
4×10^{-2}	7.753×10^{2}		

Table II

Low Wavenumber One-Dimensional Spectrum (Eq. 37)

$k\ell$	$\frac{F^1_{pp}}{\rho^2 u^4 \ell}$	$k\ell$	$\frac{F^1_{pp}}{\rho^2 u^4 \ell}$
10^{-1}	1.073×10^{-2}	6	3.894×10^{-3}
2×10^{-1}	1.073×10^{-2}	8	2.420×10^{-3}
4×10^{-1}	1.073×10^{-2}	1×10^{1}	1.606×10^{-3}
6×10^{-1}	1.073×10^{-2}	2×10^{1}	3.884×10^{-4}
8×10^{-1}	1.073×10^{-2}	4×10^{1}	8.352×10^{-5}
1	1.073×10^{-2}	6×10^{1}	3.316×10^{-5}
2	1.029×10^{-2}	8×10^{1}	1.712×10^{-5}
4	6.647×10^{-3}	10^{2}	1.022×10^{-5}

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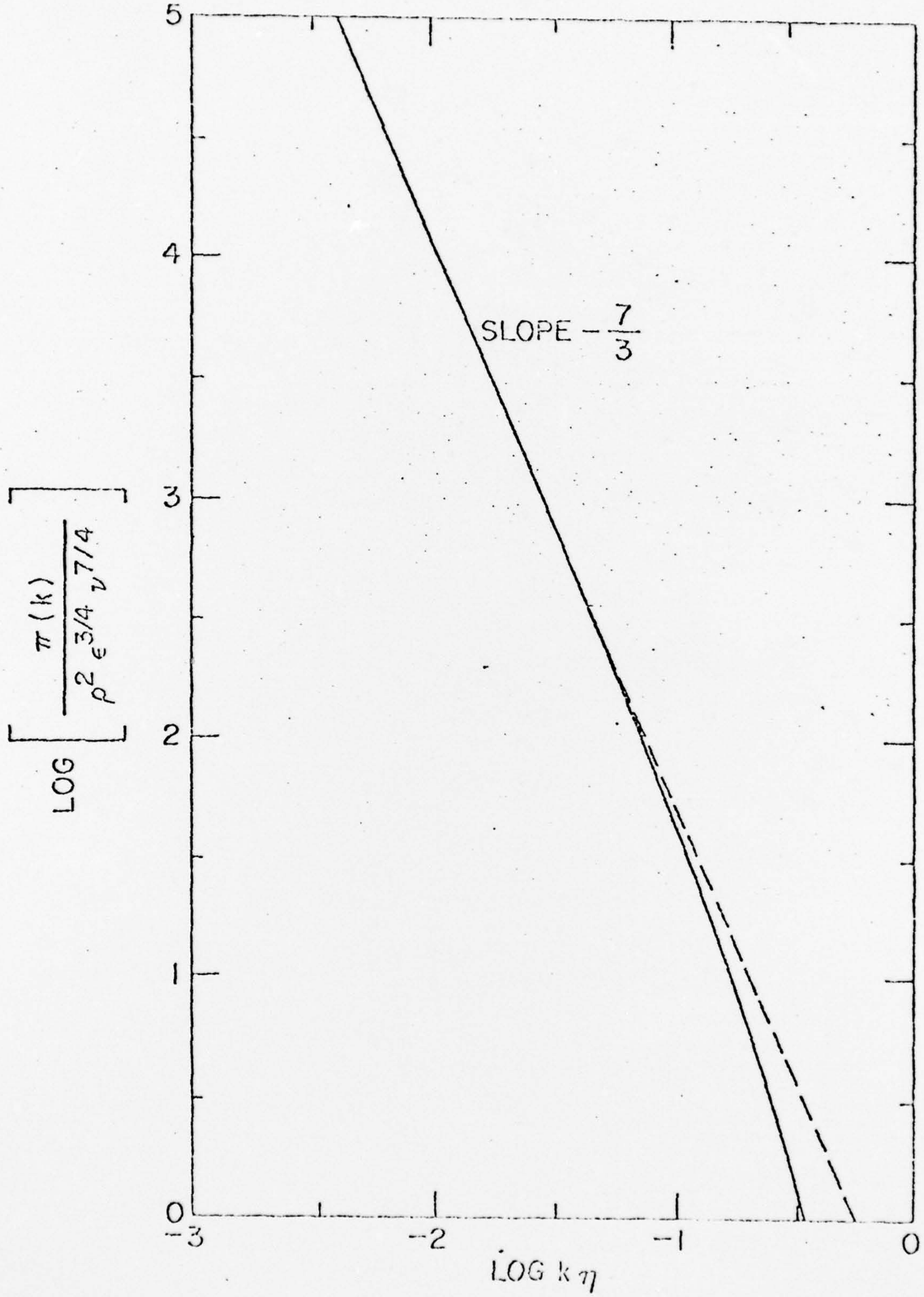

Figure 1 - High Wavenumber Spectrum from Equation (9) Using α = 1.5

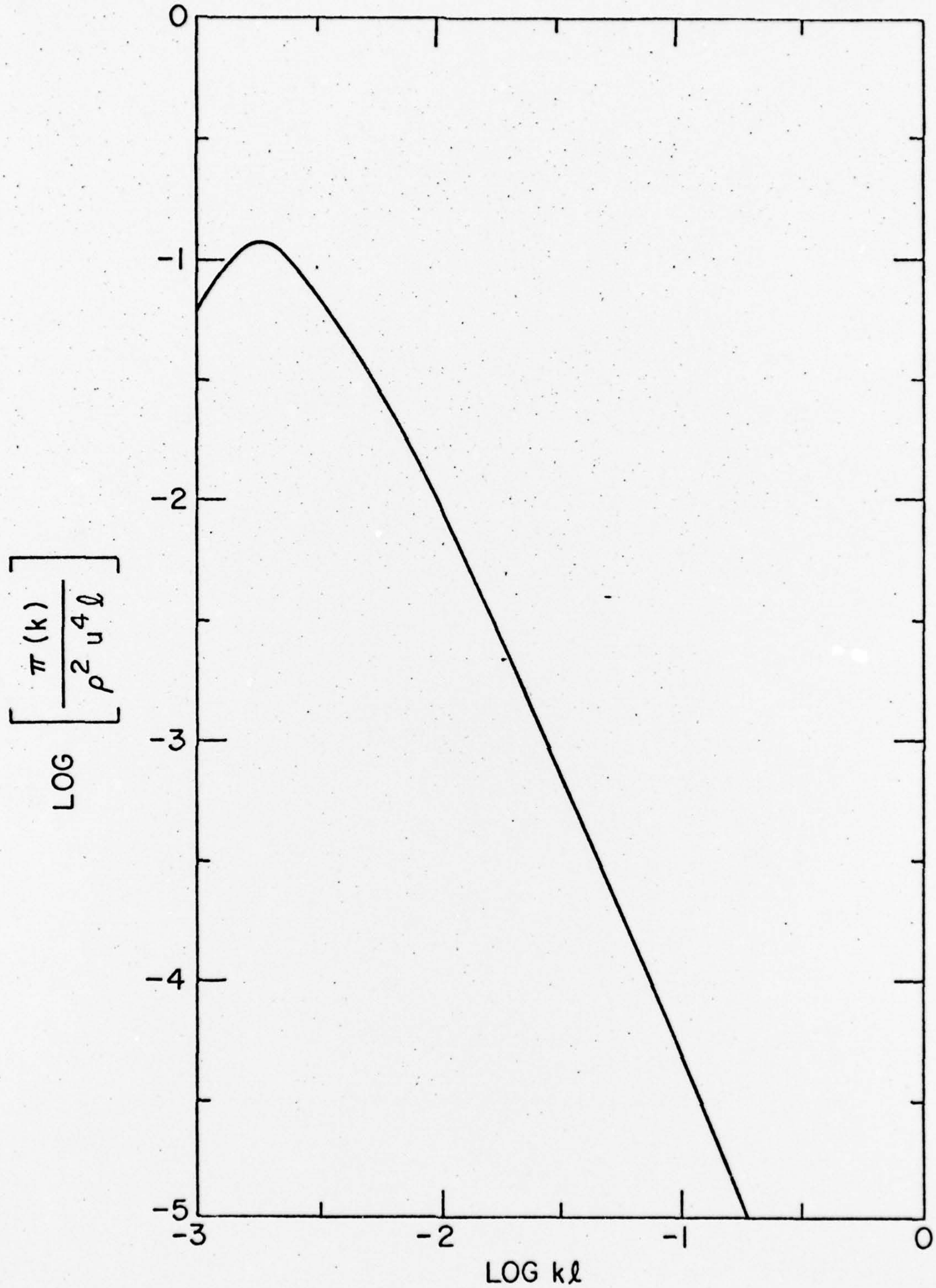

Figure 2 - Low Wavenumber Spectrum from Equation (17) Using α = 1.5

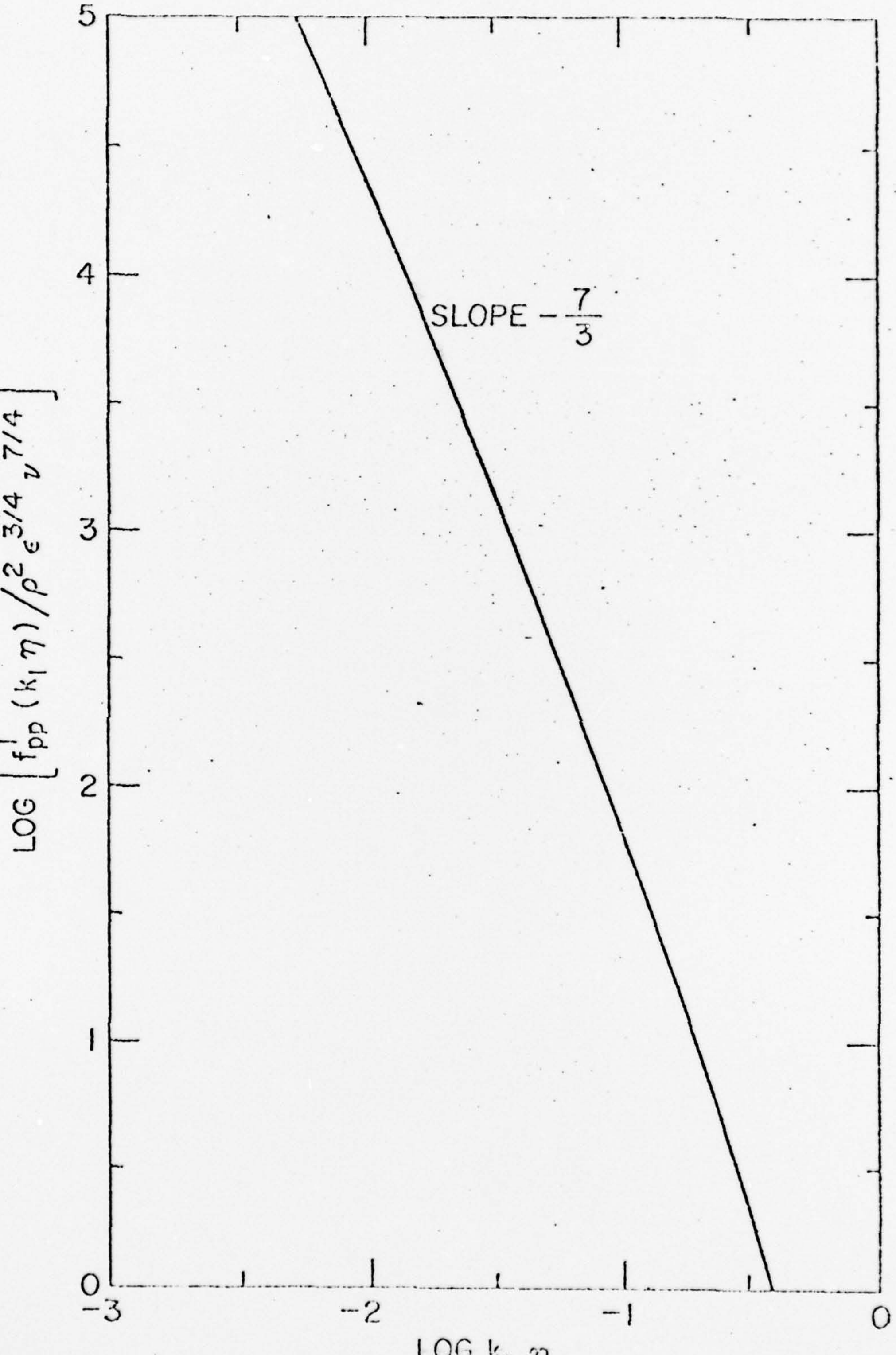

Figure 3 - High Wavenumber One-Dimensional Spectrum from Equation (36) Using α = 1.5

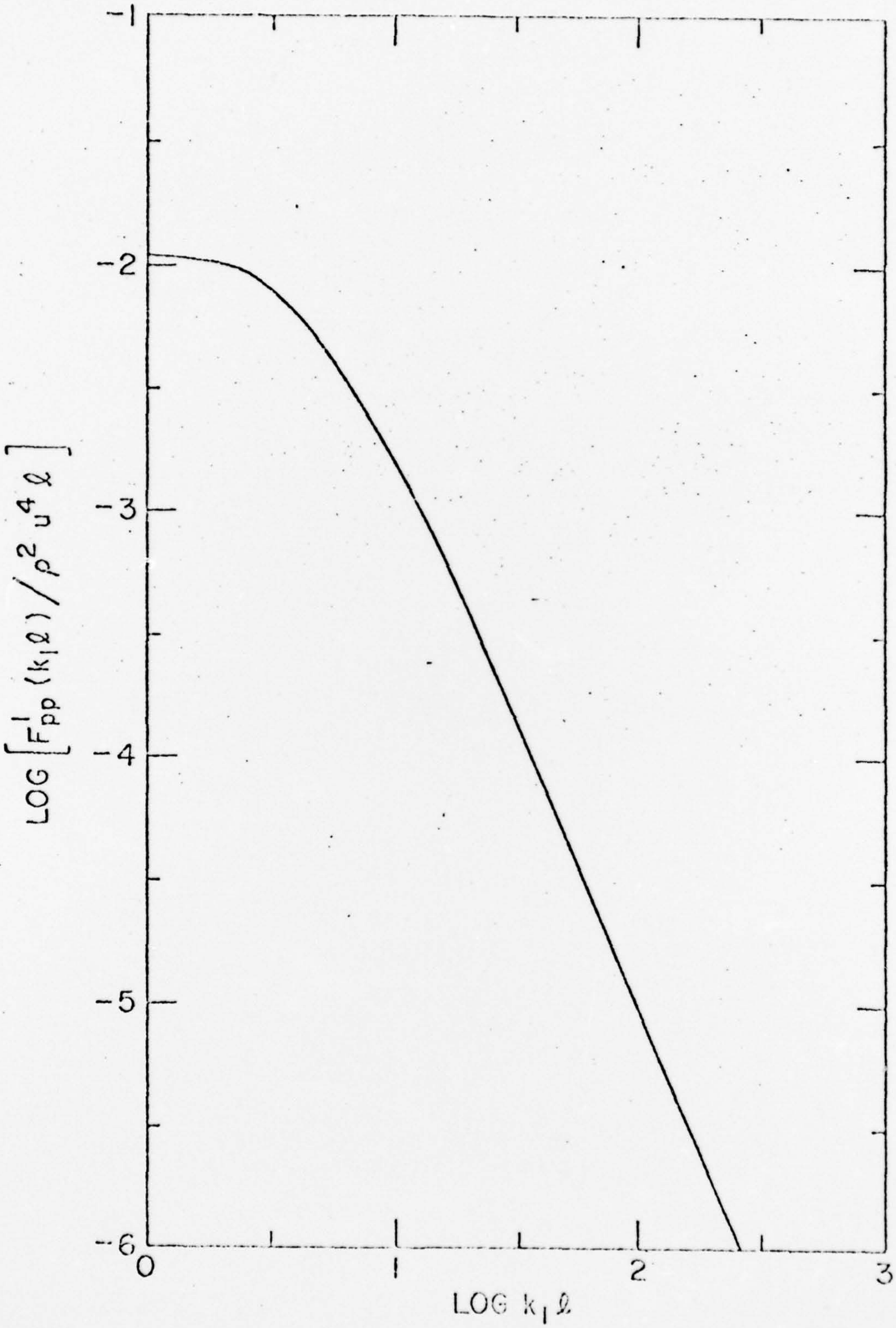

Figure 4 - Low Wavenumber One-Dimensional Spectrum from Equation (37)

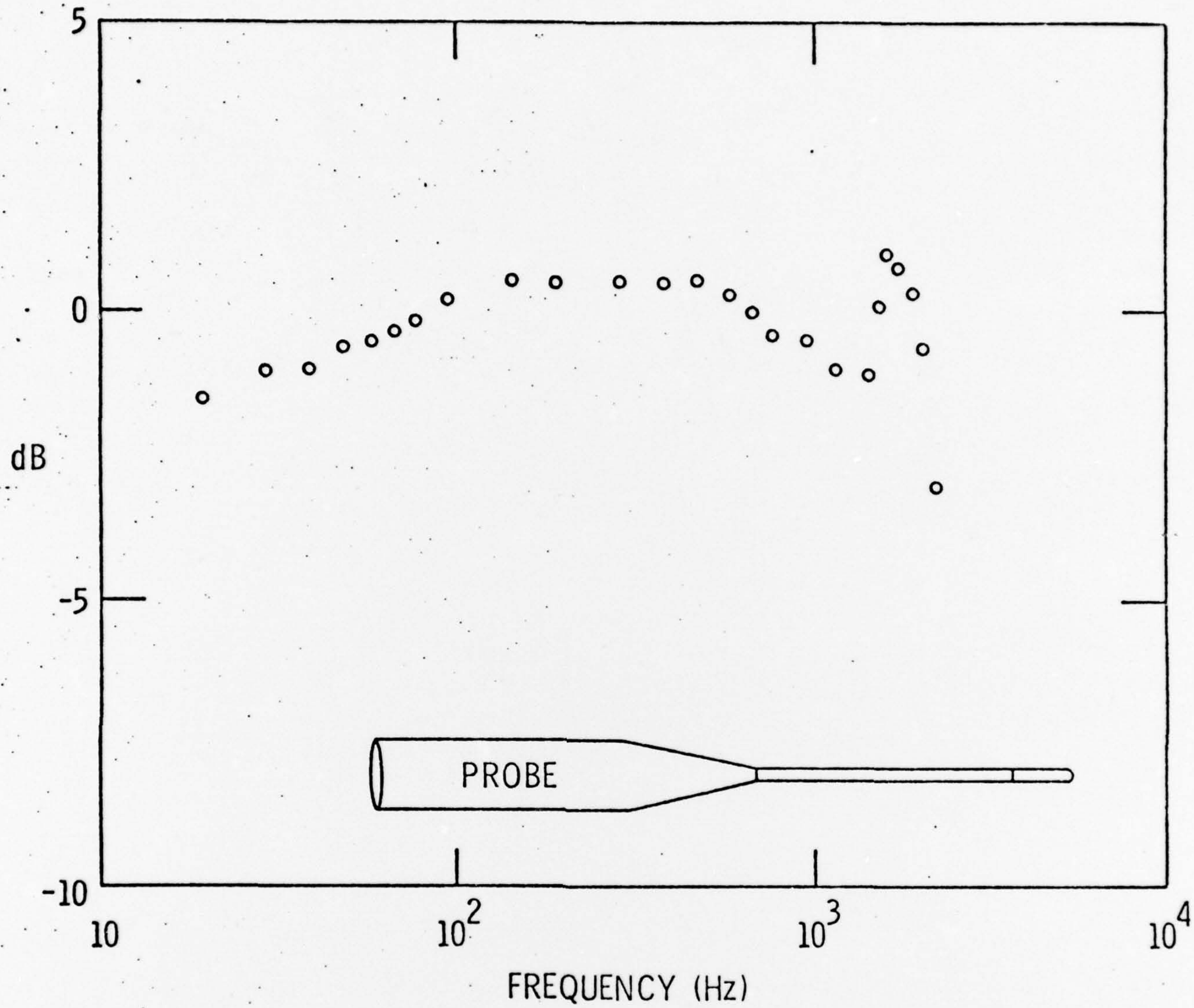

Figure 5 - Frequency Response of Pressure Probe

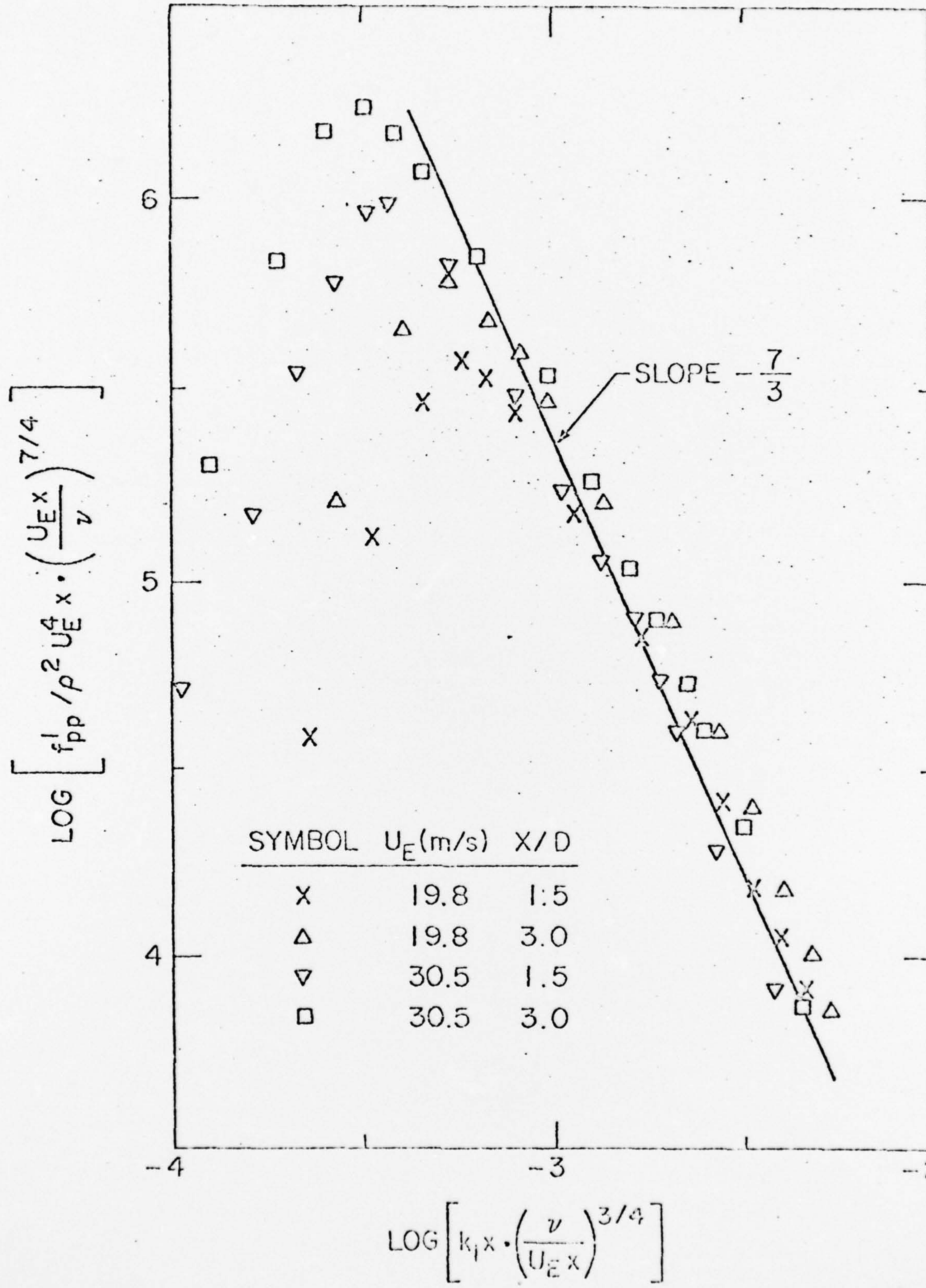

Figure 6 - Pressure Spectra Scaled with Kolmogorov-Like Variables ($\nu, \varepsilon \sim U_E^3/x$)

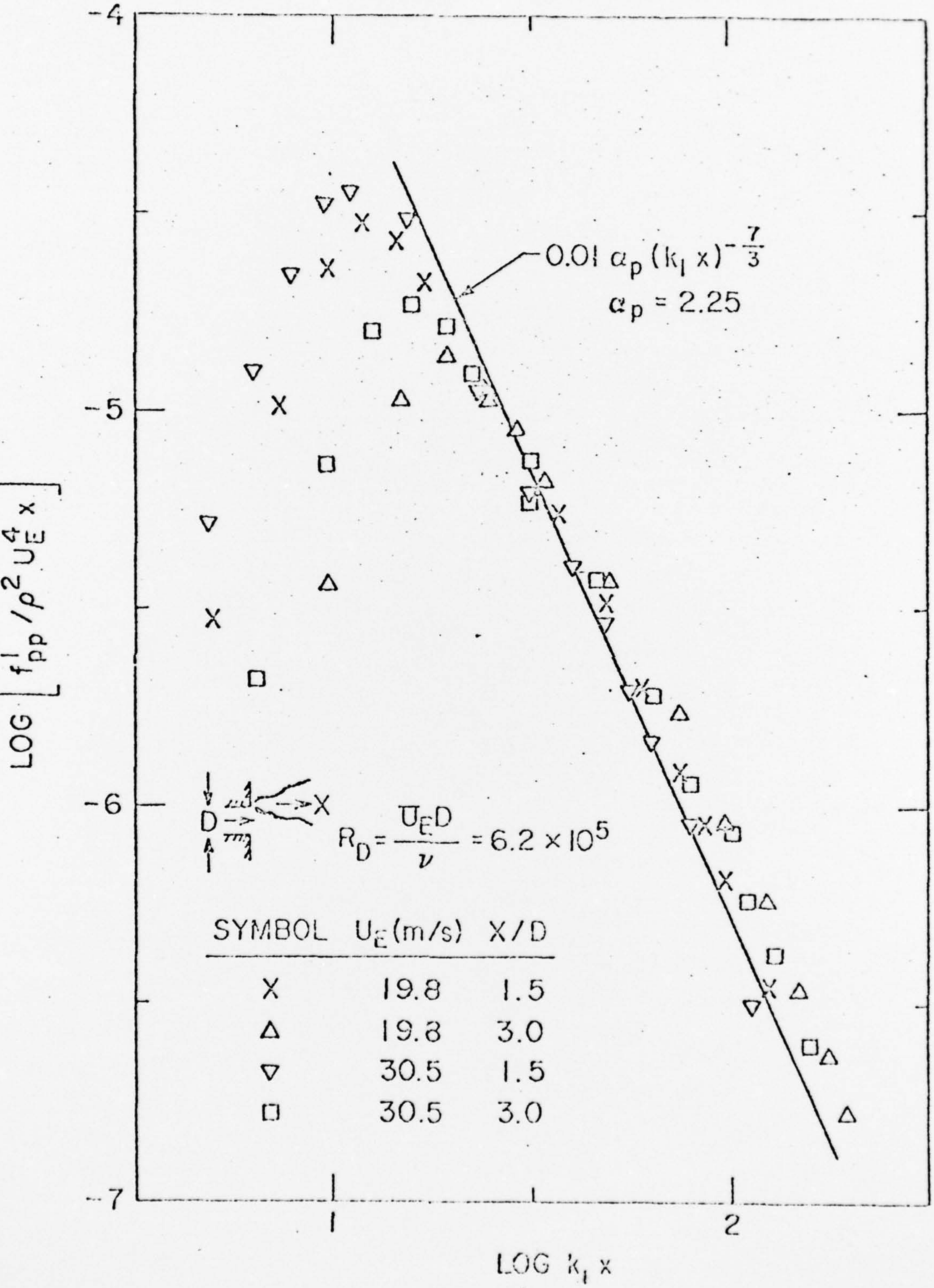

Figure 7 - Pressure Spectra Scaled with Large Scale-Like Variables (q, ℓ∼x)

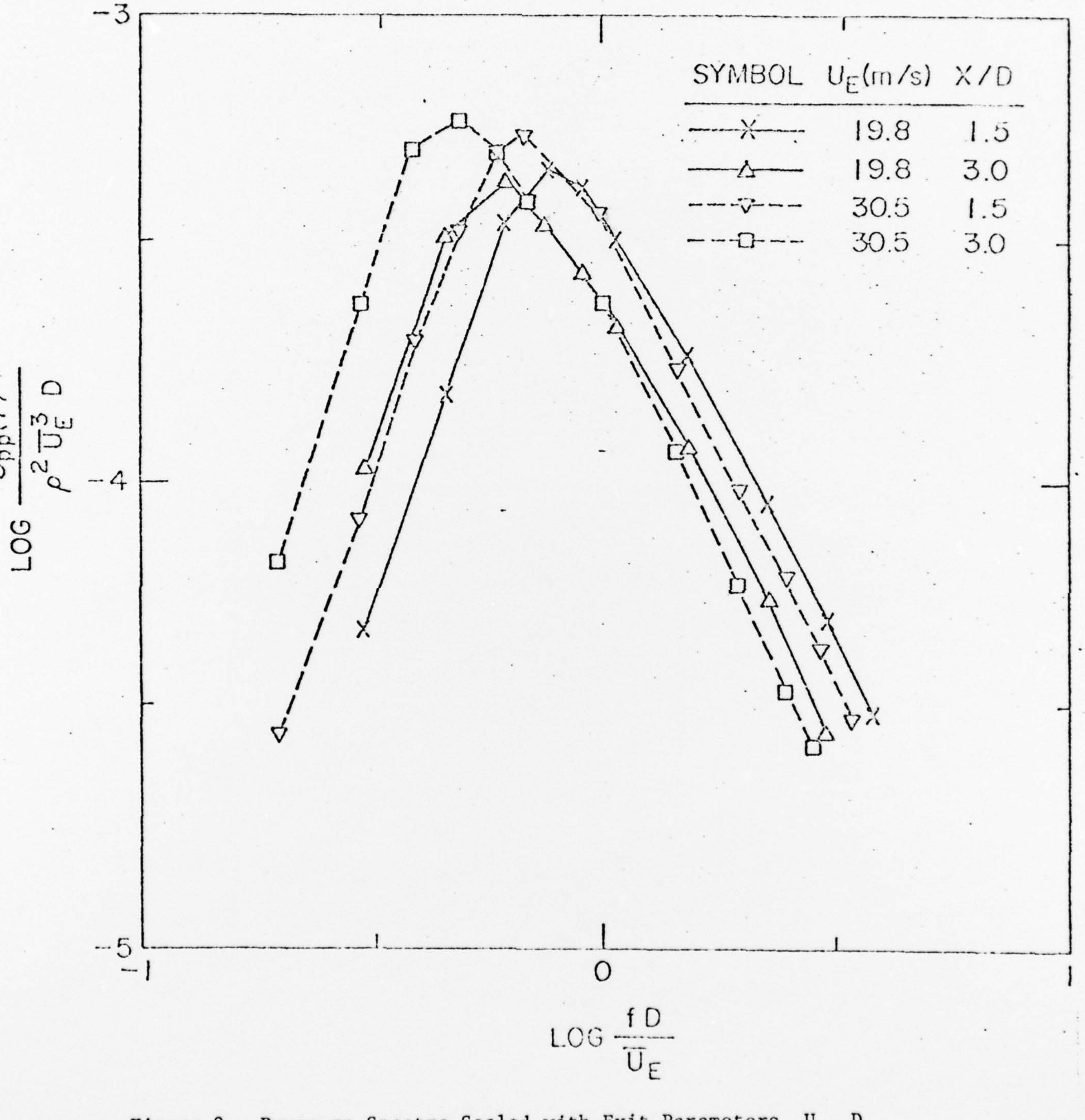

Figure 8 - Pressure Spectra Scaled with Exit Parameters, U_E, D

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State University of New York
Parker Engineering Building
Buffalo, New York 14214
(Copy No. 38)

Dr. R. E. A. Arndt
Department of Aerospace Engineering
233-L Hammond Building
The Pennsylvania State University
University Park, PA 16802
(Copy No. 39)

Dr. J. L. Lumley
Department of Aerospace Engineering
153-J Hammond Building
The Pennsylvania State University
University Park, PA 16802
(Copy No. 40)

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The Pennsylvania State University
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